

# Notes on moving mirrors

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## Abstract

The Davies-Fulling (DF) model describes the scattering of a massless field by a non-inertial mirror in two dimensions. In this paper, we generalize this model in two different ways. First, we consider partially reflecting mirrors. We show that the Bogoliubov coefficients relating inertial modes can be expressed in terms of the frequency dependent reflection factor which is specified in the rest frame of the mirror and the transformation from the inertial modes to the modes at rest with respect to the mirror. In this perspective, the DF model is simply the limiting case when this factor is unity for all frequencies. In the second part, we introduce an alternative model which is based on self-interactions described by an action principle. When the coupling is constant, this model can be solved exactly and gives rise to a partially reflecting mirror. The usefulness of this dynamical model lies in the possibility of switching off the coupling between the mirror and the field. This allows to obtain regularized expressions for the fluxes in situations where they are singular when using the DF model. Two examples are considered. The first concerns the flux induced by the disappearance of the reflection condition, a situation which bears some analogies with the end of the evaporation of a black hole. The second case concerns the flux emitted by a uniformly accelerated mirror.

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# 1 Introduction

The Davies-Fulling (DF) model [1] describes the scattering of a massless field by a non-inertial mirror in two dimensions. It has received and continues to receive attention [2, 3, 4, 5, 6, 7, 8, 9, 10, 11] principally because of its simplicity and its relationship with Hawking radiation [12]. (One can indeed mimic the varying Doppler effect induced by the collapse of a star by the total reflection on a receding mirror.) Because of its simplicity, this model has been also used to investigate the relationships between the particle description of fluxes emitted by the mirror and its field description based on the two-point Green function. The motivation behind this analysis is the following. When quantizing a field in a curved space-time one loses the uniqueness of choice for the particle notion which is then used to define the vacuum and its excitations. Based on this fact, some authors have proposed to discard the particle point of view [13]. The DF model, being defined in flat space time and giving rise to particle creation as in a curved space-time, provides a good playground for confronting the two points of view. Finally, the DF model also provides a good starting point for studying the role of ultra-high frequencies which arise in the presence of event horizons [14, 15, 16, 17]. This is particularly true when considering uniformly accelerated mirrors [3, 18, 19]. Indeed, in this case one has to confront the fact that the instantaneous value of the energy flux identically vanishes whereas the Bogoliubov coefficients, mixing positive and negative frequencies, do not vanish and lead to a total energy which furthermore diverges.

Quite independently of these specific difficulties, there is a fundamental reason which renders the analysis of these problems complicated: the DF model does not follow from an action principle. In fact, the reflection condition is imposed from the outset instead of following from interactions with the boundary. Therefore only questions concerning asymptotic properties of asymptotically inertial mirrors can be properly answered. To emphasize this point, we shall show in the first part of this article that the scattering in the DF model can be expressed in purely kinematic terms. It results from the Bogoliubov transformation relating the usual Minkowski modes to non inertial modes which are eigen-modes with respect to the proper time of the mirror. The scattering of the latter is then trivial, as trivial as the scattering of Minkowski modes by an inertial mirror. This rephrasing of the DF model is very useful in that it allows to consider partially transmitting mirrors with arbitrary frequency dependent transmission coefficients. In this perspective, the DF model is simply the limiting case in which the reflection is total for all frequencies.

In the second part of the paper we analyze an alternative model for the scattering along a given trajectory which is based on self-interactions described by an action principle. The main motivation for considering this model is that we can now switch on and off the coupling between the mirror and the field. Therefore, we can work with well defined asymptotic free states. The relationship between the partially transmitting mirrors previously considered and this model based on asymptotic states will be explicitly made.

In the first part of this Section we work with a coupling which is constant. In this case, the Born series can be exactly summed and lead to a partially transmitting mirror. Moreover, in the large coupling constant limit, one recovers the DF model, i.e. total reflection. Secondly, we consider the case when the coupling is time dependent. In this case, we compute the fluxes perturbatively to quadratic order in the coupling. The novelty arises from transient effects associated with the switching on and off. The possibility of

controlling these transients is crucial for regularizing the fluxes in situations where they are singular when using the DF model.

To make this explicit, we consider two examples. The first one consists in computing the flux associated with the appearance (or disappearance) of the reflecting boundary condition. This problem was considered by Anderson and deWitt [20]. Moreover, as discussed in [7], it presents some analogies with the residual flux associated with the disappearance of a black hole at the end of the evaporation process. When using the DF model, the flux is singular and its spectral properties are ill defined. On the contrary, with the self-interacting model, it can be described by a well-defined process in which the coupling to the mirror is switched off more and more rapidly. The second application concerns the flux emitted by a uniformly accelerated mirror. In the DF model, the energy flux vanishes everywhere but on the horizons where it is not defined. With the other model instead, a well-defined and regular expression will be obtained. In the intermediate regime we recover the vanishing of the local flux. But we also find transient effects which become singular when the switching on and off of the coupling is performed for asymptotic early and late proper times.

Finally, we wish to stress that in this paper the recoil effects shall be totally ignored since the trajectory of the mirror is given once for all. Nevertheless, since the interacting model is based on Feynman diagrams, it prepares the analysis of taking into account the dynamics of the mirror [10, 11]. Indeed the  $S$ -matrices computed with or without back-reaction effects possess a very similar structure.

## 2 The kinematic models

In the first part of this Section, we review the basic properties of the Davies-Fulling model. In particular we compare the particle description of the fluxes based on Bogoliubov coefficients with that based on the two-point functions. In the second part, we show how the scattering process can be generalized so as to describe partially transmitting mirrors. This generalization will be performed in a matrix formalism. We have chosen this formalism for two reasons: first to emphasize the kinematic nature of the DF model, and secondly to introduce in natural terms the generalization to partial reflection. In the third part, we relate the Bogoliubov coefficients to the  $S$ -matrix acting in the Fock space, thereby preparing the analysis of transition amplitudes performed in the next Section.

### 2.1 The Davies-Fulling model

In the Davies-Fulling model, the mirror is perfectly reflecting for all frequencies and its trajectory is chosen from the outset. Moreover, no width is attributed to the reflecting condition, i.e. it acts like a delta in space. Beside the fact that the trajectory is always time-like, we shall also impose that it is asymptotically inertial. In conformal terms this means that the trajectory starts from  $i^-$  and ends in  $i^+$ , the past and future time-like infinities respectively [21]. The reason is that in the other cases, i.e. when the mirror originates and/or ends on null infinities, the calculation of the energy radiated by the mirror is ill-defined. (The specific problems associated with such asymptotic trajectories will be considered in a next article [22]).

In this paper, we shall consider the scattering of a *complex* massless scalar field. The

reason for this choice is that it allows to consider more general scattering matrices when the reflection condition is not perfect. This possibility will be exploited in the next subsections. Since the dynamics of the mirror is trivial, the evolution of the field is entirely governed by the d'Alembert equation

$$(\partial_t^2 - \partial_z^2)\Phi(t, z) = 0 \quad (1)$$

and the reflection condition

$$\Phi(t, z_{cl}(t)) = 0 \quad (2)$$

along the classical trajectory of the mirror  $z = z_{cl}(t)$ .

Since the field is massless and since we work in two dimensions, it is particularly useful to work in the light-like coordinates defined by  $U, V = t \mp z$ . For instance, the general solution of eq. (1) is the sum of a function of  $U$  alone plus a function of  $V$ . Moreover, since the mirror is perfectly reflecting, the trajectory of the mirror completely decouples the left hand side configurations from the right hand side ones. Therefore, in this subsection, we can and shall restrict our attention to the configurations living on the left of the mirror.

Finally, since the mirror trajectory emerges from  $i^-$ ,  $V = -\infty$  is a complete Cauchy surface. Hence, the usual modes of the d'Alembertian given by

$$\varphi_k(U) = \frac{e^{-ikU}}{\sqrt{4\pi|k|}} \quad (3)$$

form a complete and orthonormal basis. (Instead, when the trajectory starts from the left part of  $\mathcal{J}^-$ , the choice of a complete and orthonormal basis should be reconsidered[22].) We recall that the norm of the modes is determined by the Klein-Gordon scalar product which reads, when evaluated on  $\mathcal{J}^-$  :

$$\langle \varphi_k | \varphi_{k'} \rangle = \int_{-\infty}^{+\infty} dU \varphi_k^* i \overleftrightarrow{\partial}_U \varphi_{k'} = \text{sgn}(k) \delta(k - k') . \quad (4)$$

The scattered mode corresponding to eq. (3) is determined by eq. (2) to be

$$\varphi_k^{scat}(V) = -\frac{e^{-ikU_{cl}(V)}}{\sqrt{4\pi|k|}} , \quad (5)$$

where  $U = U_{cl}(V)$  is the trajectory of the mirror in the light-like coordinates.

The *in*-mode  $\varphi_k^{in}(U, V)$  is by definition the solution of eqs. (1) and (2) which has eq. (3) as initial data. It is given by

$$\varphi_k^{in}(U, V) = \frac{e^{-ikU}}{\sqrt{4\pi|k|}} - \frac{e^{-ikU_{cl}(V)}}{\sqrt{4\pi|k|}} . \quad (6)$$

To analyze the frequency content of its scattered part, it should be Fourier decomposed on the final Cauchy surface  $U = +\infty$  (the left part of  $\mathcal{J}^+$ ). In total analogy with what we have on  $\mathcal{J}^-$ , on  $\mathcal{J}^+$  the normalized modes are

$$\varphi_\omega(V) = \frac{e^{-i\omega V}}{\sqrt{4\pi|\omega|}} . \quad (7)$$

Then the scattered mode (5) can be decomposed as

$$\varphi_k^{scat} = \int_0^\infty d\omega \left( \alpha_{\omega k}^* \varphi_\omega - \beta_{\omega k}^* \varphi_\omega^* \right) \quad (8)$$

where the coefficients  $\alpha_{\omega k}, \beta_{\omega k}$  are given by the overlaps

$$\begin{aligned} \alpha_{\omega k}^* &= \langle \varphi_\omega | \varphi_k^{scat} \rangle = -2 \int_{-\infty}^{+\infty} dV \frac{e^{i\omega V}}{\sqrt{4\pi|\omega|^{-1}}} \frac{e^{-ikU_{cl}(V)}}{\sqrt{4\pi|k|}} \\ \beta_{\omega k}^* &= \langle \varphi_\omega^* | \varphi_k^{scat} \rangle . \end{aligned} \quad (9)$$

Since both the initial and the final set of modes are complete, the coefficients  $\alpha_{\omega k}, \beta_{\omega k}$  satisfy the relations

$$\begin{aligned} \int_0^\infty dk (\alpha_{\omega k}^* \alpha_{\omega' k} - \beta_{\omega k} \beta_{\omega' k}^*) &= \delta(\omega - \omega') \\ \int_0^\infty d\omega (\alpha_{\omega k} \alpha_{\omega k'}^* - \beta_{\omega k} \beta_{\omega k'}^*) &= \delta(k - k') \\ \int_0^\infty dk (\alpha_{\omega k} \beta_{\omega k'} - \beta_{\omega k} \alpha_{\omega k'}) &= 0 \\ \int_0^\infty d\omega (\alpha_{\omega k} \beta_{\omega k'}^* - \beta_{\omega k}^* \alpha_{\omega k'}) &= 0 . \end{aligned} \quad (10)$$

Note that these relations are not trivially fulfilled when the trajectory of the mirror reaches one of the null infinities rather than the time-like ones. Notice also that the overlaps (9) can be computed on any space-like surface which runs from  $z = -\infty$  to some point on the mirror  $(t, z_{cl}(t))$ . In this case, one should use the full expression of the *in* modes given in eq. (6) as well as that of the *out* modes given by

$$\varphi_\omega^{out}(U, V) = \varphi_\omega(V) + \varphi_\omega^{bscat}(U) . \quad (11)$$

The second term  $\varphi_\omega^{bscat}$  results from the backward scattering of  $\varphi_\omega$  given in eq. (7).

When the overlaps  $\alpha_{\omega k}$  and  $\beta_{\omega k}$  are known, the classical scattering problem is solved. That is, it suffices to decompose the initial data in terms of the modes (7) to obtain, through eq. (9), the Fourier content of its image on  $\mathcal{J}^+$ . It should be pointed out that the coefficients  $\beta_{\omega k}$  which mix positive and negative frequencies have a well defined role in this classical wave theory: they determine the (non-adiabatic [23]) increase of the Fourier components of the scattered wave (see *e.g.* eq. (11) in [10] for their influence on the energy of the reflected wave). It should be also pointed out that one can recover an approximate space-time description of the scattering when considering localized wave packets rather than plane waves: for sufficiently high frequencies (i.e. short wave lengths compared to the (acceleration of the mirror) $^{-1}$ ), the coefficients  $\beta_{\omega k}$  vanish and the mean frequency of the reflected packet  $\bar{\omega}$  is related to  $\bar{k}$ , that of the incident one, by the Doppler effect  $\bar{\omega} = \bar{k} \partial_V U_{cl}|_{U=\bar{U}}$  evaluated at  $\bar{U}$ , the retarded time of the center of the incident packet. These two properties are easily obtained by evaluating the integrals in eq. (9) by the saddle point method.

When  $\alpha_{\omega k}$  and  $\beta_{\omega k}$  are known, the quantum scattering problem is also solved. This follows from the linearity of eq. (1) and eq. (2): when working in a second quantized framework, being linear, these equations provide the Heisenberg equations for the field

operator. Thus the field operator can be written both in the *in* and the *out* basis by

$$\begin{aligned}\Phi &= \int_0^\infty dk \left( a_k^{in} \varphi_k^{in} + b_k^{in\dagger} \varphi_k^{in*} \right) \\ &= \int_0^\infty d\omega \left( a_\omega^{out} \varphi_\omega^{out} + b_\omega^{out\dagger} \varphi_\omega^{out*} \right).\end{aligned}\quad (12)$$

When imposing that it satisfies the equal time commutation relation  $[\Phi(z), \partial_t \Phi^\dagger(z')] = i\delta(z - z')$ , eq. (4) guarantees that the *in*-operators  $a_k, b_k$  satisfy the usual commutation relations leading to the particle interpretation. Then, the *in* vacuum  $|0_{in}\rangle$  is defined as the product of the ground states of the *in* oscillators and its excitations are generated by the creation operators  $a_k^{in\dagger}, b_k^{in\dagger}$ . Moreover, by construction of the *in* modes on  $\mathcal{J}^-$ , the *in* particles correspond to the usual Minkowski particles on  $\mathcal{J}^-$ . Similarly, by construction of the *out* modes, all these properties apply to the *out* operators  $a_k^{out\dagger}, b_k^{out\dagger}$  and to the *out* vacuum  $|0_{out}\rangle$  when replacing  $\mathcal{J}^-$  by  $\mathcal{J}^+$ .

Given the orthonormal and complete character of the *in* and *out* mode basis, eqs. (9) and (12) determine the Bogoliubov relations :

$$\begin{cases} a_k^{in} = \int_0^\infty d\omega (\alpha_{\omega k} a_\omega^{out} + \beta_{\omega k} b_\omega^{out\dagger}) \\ b_k^{in\dagger} = \int_0^\infty d\omega (\beta_{\omega k}^* a_\omega^{out} + \alpha_{\omega k}^* b_\omega^{out\dagger}) \end{cases} \quad \begin{cases} a_\omega^{out} = \int_0^\infty dk (\alpha_{\omega k}^* a_k^{in} - \beta_{\omega k} b_k^{in\dagger}) \\ b_\omega^{out\dagger} = \int_0^\infty dk (-\beta_{\omega k}^* a_k^{in} + \alpha_{\omega k} b_k^{in\dagger}). \end{cases} \quad (13)$$

Then eqs. (10) guarantee the compatibility of the particle interpretation in each basis, i.e., both *in* and *out* operators obey the canonical commutations relations. With the relations (13), all questions concerning quantum scattering processes can be answered. For instance, the probability amplitude to obtain a given final state  $|\Psi_{fin}\rangle$  specified on  $\mathcal{J}^+$  in terms of *out* operators starting from some *in* state  $|\Xi_{in}\rangle$  constructed on  $\mathcal{J}^-$  is given by the (Fock space) product  $\langle \Psi_{fin} | \Xi_{in} \rangle$ . More intrinsic is the overlap  $Z^{-1} = \langle 0_{out} | 0_{in} \rangle$  between the initial and final vacuum states. Indeed, it determines the probability amplitude for the (spontaneous) decay of the vacuum. The computation of  $Z$  is easy when the scattering is stationary, see *e.g.* [2, 8]. In the general case however, due to the frequency mixing between *in* and *out* modes, the calculation of  $Z$  is less trivial. This generalization is presented in Appendix A.

It should also be noted that the Bogoliubov coefficients themselves are given by the following matrix elements

$$\begin{aligned}\alpha_{\omega k}^* &= \langle 0_{in} | a_\omega^{out} a_k^{in\dagger} | 0_{in} \rangle, \\ -\beta_{\omega k}^* &= \langle 0_{in} | b_\omega^{out\dagger} a_k^{in\dagger} | 0_{in} \rangle.\end{aligned}\quad (14)$$

However it is not clear how to attribute a physical meaning to these equations. In particular, the relationship with the second one and pair creation amplitude is quite obscure. Indeed the *probability* amplitude to obtain on  $\mathcal{J}^+$  one pair of quanta of frequencies  $\omega$  and  $\omega'$  in the *in* vacuum is given by

$$\langle 0_{out} | a_\omega^{out} b_{\omega'}^{out} | 0_{in} \rangle = -\frac{1}{Z} \int_0^\infty dk \beta_{\omega k} \alpha_{k\omega'}^{-1}. \quad (15)$$

We shall return to these questions of interpretation in subsection 2.3.

Instead of considering *in* – *out* matrix elements in Fock space, more attention has been put on expectation values of (local) operators in a given initial state. The most

studied object is probably the energy flux emitted by the mirror when the state of the field is *in* vacuum. The motivations for this analysis are, first, its relevance for black hole radiation[2]-[11], secondly, that its non-vanishing value is due to spontaneous pair creation, a specific feature of quantum field theory (QFT), and thirdly that this value can be computed both from using eqs. (13) and from the properties of the Green function of the field.

Having at our disposal the Bogoliubov coefficients  $\alpha_{\omega k}, \beta_{\omega k}$ , we start with the particle point of view. We consider the density energy of the emitted flux. The corresponding hermitian operator is<sup>3</sup>  $T_{VV} = \partial_V \Phi^\dagger \partial_V \Phi + \partial_V \Phi \partial_V \Phi^\dagger$ . On the left of the mirror ( $U > U_{cl}(V)$ ), using eq. (13) and the first line of eq. (10), the expectation value of the energy flux is

$$\begin{aligned} \langle T_{VV} \rangle &\equiv \langle 0_{in} | T_{VV} | 0_{in} \rangle - \langle 0_{out} | T_{VV} | 0_{out} \rangle \\ &= 2 \Re e \left[ \iint_0^\infty d\omega d\omega' \frac{\sqrt{\omega\omega'}}{2\pi} \right. \\ &\quad \left. \left[ e^{-i(\omega'-\omega)V} \left( \int_0^\infty dk \beta_{\omega k}^* \beta_{\omega' k} \right) - e^{-i(\omega'+\omega)V} \left( \int_0^\infty dk \alpha_{\omega k}^* \beta_{\omega' k} \right) \right] \right]. \end{aligned} \quad (16)$$

It should be noted that the subtraction of the *out* vacuum flux follows from the prescription of subtracting the contribution of the Minkowski vacuum. Indeed, by construction of the *out* modes, they coincide with the usual Minkowski modes on  $\mathcal{J}^+$ .

The total energy emitted to  $\mathcal{J}^+$  is obtained from integrating  $\langle T_{VV} \rangle$  over  $V$ . The integration eliminates the second term which is due to interferences between states with different particle numbers. It gives

$$\begin{aligned} \langle H_V \rangle &= \int_{-\infty}^{+\infty} dV \langle T_{VV} \rangle \\ &= 2 \int_0^\infty d\omega \omega \int_0^\infty dk |\beta_{\omega k}|^2 = 2 \int_0^\infty d\omega \omega \langle n_\omega \rangle. \end{aligned} \quad (17)$$

One gets the usual relationship between the mean energy and the mean number of particles  $\langle n_\omega \rangle = \int_0^\infty dk |\beta_{\omega k}|^2$  found on  $\mathcal{J}^+$  (it equals to the number of anti-particles). In this writing one sees that the non-vanishing character of  $\langle H_V \rangle$  is due to the  $\beta$  coefficients which govern the vacuum decay, see eq. (109) in App. A.

We now reconsider the flux  $\langle T_{VV} \rangle$  without making use of the Bogoliubov coefficients and with less emphasis on the notion of particle. This method is based on the two-point Wightman function evaluated in the *in* vacuum

$$\langle 0_{in} | \Phi^\dagger(U, V) \Phi(U', V') | 0_{in} \rangle = \int_0^\infty dk \varphi_k^{in}(U, V) \varphi_k^{in*}(U', V'). \quad (18)$$

In terms of this function, using eq. (5), the mean flux on  $\mathcal{J}^+$  reads

$$\begin{aligned} \langle T_{VV} \rangle &= 2 \lim_{V' \rightarrow V} \left[ \langle 0_{in} | \partial_V \Phi^\dagger \partial_{V'} \Phi | 0_{in} \rangle - \langle 0_{out} | \partial_V \Phi^\dagger \partial_{V'} \Phi | 0_{out} \rangle \right] \\ &= -\frac{1}{2\pi} \lim_{V' \rightarrow V} \partial_V \partial_{V'} [\ln |U_{cl}(V') - U_{cl}(V)| - \ln |V' - V|] \end{aligned}$$

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<sup>3</sup> The symmetrization is due to the fact that we deal with a complex field. Of course, in the DF model, particles and anti-particles equally contribute to  $\langle T_{VV} \rangle$ . This explains the overall factors of 2 in the next equations. We warn the reader that this equal contribution will not be necessarily found when considering partially transmitting mirrors.

$$\begin{aligned}
&= \frac{1}{6\pi} \left[ \left( \frac{dU_{cl}}{dV} \right)^{1/2} \partial_V^2 \left( \left( \frac{dU_{cl}}{dV} \right)^{-1/2} \right) \right] \\
&= \frac{1}{24\pi} \left[ \left( \frac{d^2 U_{cl}}{dV^2} \right) \left( \frac{dU_{cl}}{dV} \right)^{-1} \right]^2 - \frac{1}{12\pi} \partial_V \left[ \left( \frac{d^2 U_{cl}}{dV^2} \right) \left( \frac{dU_{cl}}{dV} \right)^{-1} \right]. \quad (19)
\end{aligned}$$

Again, the subtraction of the *out* vacuum flux follows from the prescription of subtracting the contribution of the Minkowski vacuum. In this second description of the flux, it is through this prescription that the notion of vacuum decay is re-introduced. Indeed, on  $\mathcal{J}^+$ , the above subtraction is equivalent to normal ordering with respect to *out* operators. (This is straightforwardly proven by using eq. (10).) Moreover, the fact that  $\langle T_{VV} \rangle$  identically vanishes for inertial trajectories, i.e. when  $\partial_V^2 U_{cl} = 0$ , confirms that its non-vanishing character is due to the non-adiabaticity[23] of the scattering, a notion deeply rooted to the spontaneous creation of pair of particles.

From eq. (19) we learn that the energy flux is local in that it depends only on three derivatives of the trajectory  $U_{cl}(V)$  evaluated at the retarded time  $V$  (remember that we are on the left of the mirror). We shall see below that this locality is a consequence of dealing with a perfectly reflecting mirror for all frequencies.

Notice finally that in eq. (16), the first term is positive definite and leads to the positive total energy  $\langle H_V \rangle$ . Indeed, being a total derivative, the second term does not contribute to  $\langle H_V \rangle$  when the trajectory is asymptotically inertial since  $U_{cl}(V) \sim V$  for asymptotically late and early  $V$ 's. This might not be the case for trajectories which enter or leave the space through the null infinities, because of the infinite Doppler effect encountered asymptotically.

## 2.2 Partially transmitting mirrors

In this subsection we study partially transmitting (but still recoil-less) non-inertial mirrors. We shall proceed in three steps. We first show that the scattering by a non-inertial mirror is most simply described in terms of the wave functions which are eigen-modes of the proper time of the mirror. (We shall call them the proper-time modes.) When using these modes, the matrix relating the scattered modes to the initial ones is diagonal in the frequency, exactly like for the scattering of Minkowski modes by a mirror at rest. Secondly, we shall see that these modes are well adapted to introduce partially reflecting coefficients with arbitrary frequency dependent phase and amplitude. Indeed, since this matrix is diagonal in the proper-time frequency, unitarity constraints its elements in a simple manner, frequency by frequency. Thirdly, for both partial and total reflection, we shall see that the usual Bogoliubov coefficients, eq. (9), relating the *in* and *out* Minkowski modes are simply obtained from this diagonal matrix.

To fulfill this program, we first need to construct the proper-time modes. To this end it is very useful to introduce new light-like coordinates  $u, v$  such that the time-like coordinate  $(u+v)/2 = \tau$  is the proper time of the mirror and the space-like one defined by  $(v-u)/2 = \rho$  is such that the trajectory reads  $\rho = \rho_0 = \text{constant}$ . These new coordinates are defined by two analytic functions  $u(U)$  and  $v(V)$  where  $U, V$  are the Minkowski light-like coordinates. These functions are determined by the mirror trajectory  $U_{cl}(V)$  and the two conditions defining  $\tau$  and  $\rho$ . Indeed, along the mirror's trajectory, the length element



obeys

$$\begin{aligned} ds^2 &= \partial_V U_{cl}(V) dV^2 = \partial_U V_{cl}(U) dU^2 \\ &= dv^2 = du^2 = d\tau^2 . \end{aligned} \quad (20)$$

This gives

$$\frac{dv}{dV} = \sqrt{\partial_V U_{cl}} \quad \text{and} \quad \frac{du}{dU} = \sqrt{\partial_U V_{cl}} . \quad (21)$$

One verifies that the Jacobians  $dv/dV$  and  $du/dU$  define a time dependent boost since they satisfy  $(dv/dV)(du/dU) = 1$  for all  $\tau$ . The proper-time modes are then simply given by

$$\begin{aligned} \varphi_\lambda(u) &= \frac{e^{-i\lambda u}}{\sqrt{4\pi|\lambda|}} , \\ \varphi_\lambda(v) &= \frac{e^{-i\lambda v}}{\sqrt{4\pi|\lambda|}} . \end{aligned} \quad (22)$$

They form a complete basis on  $\mathcal{J}^-$  and  $\mathcal{J}^+$  since our condition that the trajectory emerges from  $i^-$  and finishes on  $i^+$  implies that the  $v$  and  $u$  axes cover those of  $V$  and  $U$  respectively.

In the case of total reflection, the scattering along the mirror at  $\rho = \rho_0$  is trivial. When using the conventions of the former subsection (eqs. (6) and (11)), one has, on the left of the mirror,

$$\begin{aligned} \varphi_\lambda^{U,in}(u, v) &= \varphi_\lambda(u) - e^{2i\lambda\rho_0} \varphi_\lambda(v) \\ &= -e^{2i\lambda\rho_0} \varphi_\lambda^{V,out}(u, v) . \end{aligned} \quad (23)$$

The new subscript  $U, V$  indicates which side of  $\mathcal{J}^-$  ( $\mathcal{J}^+$ ) is the asymptotic support of the *in* (*out*) functions. We have introduced it in order to describe partial reflection which requires to consider simultaneously both sides of the mirror. Using this notation, on the right of the mirror, one has

$$\begin{aligned} \varphi_\lambda^{V,in}(u, v) &= \varphi_\lambda(v) - e^{-2i\lambda\rho_0} \varphi_\lambda(u) \\ &= -e^{-2i\lambda\rho_0} \varphi_\lambda^{U,out}(u, v) . \end{aligned} \quad (24)$$

It will be useful to express these relations by a  $2 \times 2$  matrix  $\mathbf{S}_\lambda$  as

$$\varphi_\lambda^{i,out} = S_\lambda^{ij} \varphi_\lambda^{j,in} \quad \left( \equiv a_\lambda^{j,in} = S_\lambda^{ij} a_\lambda^{i,out} \right) . \quad (25)$$

At fixed  $\lambda$ , the indices of rows and columns  $i, j$  are the new subscript  $U$  or  $V$ . As usual, repeated indices are summed over. For total reflection, one has

$$\mathbf{S}_\lambda = \begin{pmatrix} 0 & -e^{+2i\lambda\rho_0} \\ -e^{-2i\lambda\rho_0} & 0 \end{pmatrix} . \quad (26)$$

We now consider partial reflection. When considering elastic reflection, the matrix  $\mathbf{S}_\lambda$  relating *in* and *out* modes which generalizes eq. (25) is unitary. (That is, we generalize

total reflection in a restricted way since we keep both the linearity and the unitarity of eq. (25).) Unitarity constraints the elements of  $\mathbf{S}_\lambda$

$$\mathbf{S}_\lambda = \begin{pmatrix} s_u e^{i\varphi_u} & -i R e^{i\varphi} \\ -i R' e^{i\varphi'} & s_v e^{i\varphi_v} \end{pmatrix} \quad (27)$$

to obey

$$\begin{aligned} R &= R' \quad , \quad s_u = s_v \quad , \\ s_u^2 + R^2 &= 1 \quad \text{and} \quad \varphi' = \varphi_u + \varphi_v - \varphi \quad . \end{aligned} \quad (28)$$

(For simplicity of the expressions, we have not written the argument  $\lambda$  but all variables should be understood as  $\lambda$  dependent.) Physically,  $R$  and  $s$  correspond to the reflection and transmission coefficients, *i.e.* when working in the rest frame of the mirror, the probability for an incident quantum of frequency  $\lambda$  to be reflected is  $R^2$ .

In what follows we impose  $\varphi_u = \varphi_v = -\phi$ , a condition which expresses that the transmitted part of scattering is independent of the sign of the momentum. In anticipation to Section 3, we point out that this equality is automatically satisfied when considering parity invariant hamiltonians (see [24] Chap. 3.4). In this case the matrix reads

$$\mathbf{S}_\lambda = e^{-i\phi} \begin{pmatrix} \sqrt{1-R^2} & -i R e^{i\theta} \\ -i R e^{-i\theta} & \sqrt{1-R^2} \end{pmatrix} \quad (29)$$

In principle the common phase  $e^{-i\phi}$  could be re-absorbed in a redefinition of the modes. However, when using *in* and *out* modes conventionally defined, *i.e.*  $\varphi_\lambda^{V,in}(v) = \varphi_\lambda^{V,out}(v) = \varphi_\lambda(v)$  of eq. (22), the phase  $\phi$  is univocally fixed. As we shall see in the next Section, this convention is automatically used when considering interactions perturbatively. This is also the case in the DF model. Indeed the limiting case of total reflection given in eq. (26) is reached for  $R \rightarrow 1$  and  $\phi = \pi/2$  for all  $\lambda$ . One also finds that the other phase  $\theta$  is related to the mirror location by  $\theta = 2\rho_0\lambda$ .

To complete our second step, we should describe particles and anti-particles simultaneously. To this end, we group the *in* operators ( $a_\lambda^{U,in}, a_\lambda^{V,in}, b_\lambda^{U,in\dagger}, b_\lambda^{V,in\dagger}$ ) in a 4-vector  $a_\lambda^{\mu,in}$  and the *out* operators ( $a_\lambda^{U,out}, a_\lambda^{V,out}, b_\lambda^{U,out\dagger}, b_\lambda^{V,out\dagger}$ ) in  $a_\lambda^{\mu,out}$ . Similarly, we group their corresponding modes in the 4-vectors  $\varphi_\lambda^{\mu,in}$  and  $\varphi_\lambda^{\mu,out}$ . Since we work with a charged field, the modes associated with  $b_\lambda^{i\dagger}$  might not be the complex conjugates of those associated with  $a_\lambda^i$ . (As it is the case when dealing with a charged field in an electro-magnetic field, see *e.g.* Sect. 1.3 in [8]). Explicitly, in our case, the 4 modes are  $(\varphi_\lambda^U, \varphi_\lambda^V, \bar{\varphi}_\lambda^{U*}, \bar{\varphi}_\lambda^{V*})$  where  $\bar{\varphi}_\lambda^i$  designate the two modes associated with anti-particles operators  $b_\lambda^U$  and  $b_\lambda^V$ .

We then introduce the  $4 \times 4$  matrix given by

$$\mathbf{S}_{\lambda\lambda'} = \delta(\lambda - \lambda') \begin{pmatrix} \mathbf{S}_\lambda & 0 \\ 0 & \bar{\mathbf{S}}_\lambda^* \end{pmatrix} \quad (30)$$

where  $\bar{\mathbf{S}}_\lambda$  is the scattering matrix for the anti-particles. Since  $\mathbf{S}_{\lambda\lambda'}$  is block diagonal, unitarity constrains  $\mathbf{S}_\lambda$  and  $\bar{\mathbf{S}}_\lambda$  separately.  $\mathbf{S}_{\lambda\lambda'}$  acts on the *in* 4-vector as follows

$$\varphi_\lambda^{\mu,out} = S_{\lambda\lambda'}^{\mu\nu} \varphi_{\lambda'}^{\nu,in} \quad (31)$$

where continuous repeated indices are integrated from  $0 \rightarrow \infty$  and discrete ones summed over the four components defined at fixed frequency. With these choices, the components

of  $\mathbf{S}_{\lambda\lambda'}$  are the Bogoliubov coefficients conventionally defined. By conventionally defined we mean the equations which generalize eq. (13), i.e.,

$$\begin{aligned} a_{\lambda'}^{j,in} &= \alpha_{\lambda\lambda'}^{ij} a_{\lambda}^{i,out} + \bar{\beta}_{\lambda\lambda'}^{ij} b_{\lambda}^{i,out\dagger}, \\ b_{\lambda'}^{j,in\dagger} &= \beta_{\lambda\lambda'}^{ij*} a_{\lambda}^{i,out} + \bar{\alpha}_{\lambda\lambda'}^{ij*} b_{\lambda}^{i,out\dagger}, \end{aligned} \quad (32)$$

where the Bogoliubov coefficients  $\alpha, \bar{\alpha}, \beta, \bar{\beta}$  are now  $2 \times 2$  matrices. By direct identification, one obtains

$$\begin{aligned} \alpha_{\lambda\lambda'}^{ij} &= \langle \varphi_{\lambda'}^{j,in} | \varphi_{\lambda}^{i,out} \rangle = S_{\lambda\lambda'}^{ij}, \\ \bar{\alpha}_{\lambda\lambda'}^{ij} &= \langle \bar{\varphi}_{\lambda'}^{j,in} | \bar{\varphi}_{\lambda}^{i,out} \rangle = S_{\lambda\lambda'}^{i+2 \ j+2*}, \\ \beta_{\lambda\lambda'}^{ij} &= \langle \bar{\varphi}_{\lambda'}^{j,in} | \varphi_{\lambda}^{i,out*} \rangle = S_{\lambda\lambda'}^{i \ j+2*}, \\ \bar{\beta}_{\lambda\lambda'}^{ij} &= \langle \varphi_{\lambda'}^{j,in} | \bar{\varphi}_{\lambda}^{i,out*} \rangle = S_{\lambda\lambda'}^{i+2 \ j}. \end{aligned} \quad (33)$$

When  $\mathbf{S}_{\lambda\lambda'}$  is block-diagonal in the sense of eq. (30), one obviously has  $\beta_{\lambda\lambda'}^{ij} = \bar{\beta}_{\lambda\lambda'}^{ij} = 0$ . In full generality,  $\mathbf{S}_{\lambda\lambda'}$  satisfies unitarity in the following sense

$$(S^\dagger)^{\mu\nu}_{\lambda\lambda'}, S^{\mu'\nu'}_{\lambda'\lambda} = \delta(\lambda - \lambda') \delta^{\mu\mu'}. \quad (34)$$

This equation generalizes eqs. (10) to partially transmitting mirrors.

With eqs. (26), (30) and (31), we have shown that the scattering in the DF model is trivial when using the proper-time modes. We have done more since eqs. (31) and (33) apply to all partially transmitting mirrors governed by  $\mathbf{S}_{\lambda}$  given by eq. (29).

The last step consists in finding the relationship between  $\mathbf{S}_{\lambda\lambda'}$  and the Bogoliubov coefficients between *in* and *out* Minkowski modes. This is simply achieved by introducing the  $4 \times 4$  matrix which relates the (unscattered) Minkowski modes of frequency  $k = -i\partial_t$  to the (unscattered) proper time modes of frequency  $\lambda = -i\partial_\tau$ :

$$\phi_k^\mu = \mathcal{B}_{k\lambda}^{\mu\nu} \varphi_\lambda^\nu. \quad (35)$$

The elements of this matrix are given by

$$\begin{aligned} \mathcal{B}_{k\lambda} &= \begin{pmatrix} \langle \varphi_\lambda^U | \phi_k^U \rangle & 0 & -\langle \varphi_\lambda^{U*} | \phi_k^U \rangle & 0 \\ 0 & \langle \varphi_\lambda^V | \phi_k^V \rangle & 0 & -\langle \varphi_\lambda^{V*} | \phi_k^V \rangle \\ \langle \varphi_\lambda^U | \phi_k^{U*} \rangle & 0 & -\langle \varphi_\lambda^{U*} | \phi_k^{U*} \rangle & 0 \\ 0 & \langle \varphi_\lambda^V | \phi_k^{V*} \rangle & 0 & -\langle \varphi_\lambda^{V*} | \phi_k^{V*} \rangle \end{pmatrix} \\ &= \begin{pmatrix} \alpha_{k\lambda}^{UU} & 0 & \beta_{k\lambda}^{UU*} & 0 \\ 0 & \alpha_{k\lambda}^{VV} & 0 & \beta_{k\lambda}^{VV*} \\ \beta_{k\lambda}^{UU} & 0 & \alpha_{k\lambda}^{UU*} & 0 \\ 0 & \beta_{k\lambda}^{VV} & 0 & \alpha_{k\lambda}^{VV*} \end{pmatrix}. \end{aligned} \quad (36)$$

Since  $\mathcal{B}_{k\lambda}$  relates unscattered modes, it is independent of the charge of the particle, hence  $\mathcal{B}_{k\lambda}^{11} \equiv \alpha_{k\lambda}^{UU} = \bar{\alpha}_{k\lambda}^{UU} \equiv \mathcal{B}_{k\lambda}^{33}$ . The same equality applies to  $\alpha_{k\lambda}^{VV}, \beta_{k\lambda}^{VV}$ , and  $\beta_{k\lambda}^{UU}$ .

The important point for us is that  $\mathcal{B}_{k\lambda}$  also relates the *in* Minkowski modes to the *in* proper-time modes and the *out* Minkowski modes to the *out* proper-time modes. Therefore the linear relation between *in* and *out* Minkowski modes is given by

$$\phi_\omega^{\mu,out} = S_\omega^{\mu\mu'} \phi_k^{\mu',in} \quad (37)$$

where

$$S_{\omega k}^{\mu\mu'} = \mathcal{B}_{\omega\lambda}^{\mu\nu} S_{\lambda\lambda'}^{\nu\nu'} (\mathcal{B}_{k\lambda'}^{-1})^{\nu'\mu'}. \quad (38)$$

Repeated indices are summed over and the inverse of  $\mathcal{B}$  is defined by

$$\varphi_\lambda^\nu = (\mathcal{B}_{k\lambda}^{-1})^{\nu\mu} \phi_k^\mu. \quad (39)$$

It is given by

$$\mathcal{B}_{k\lambda}^{-1} = \begin{pmatrix} \alpha_{k\lambda}^{UU*} & 0 & -\beta_{k\lambda}^{UU*} & 0 \\ 0 & \alpha_{k\lambda}^{VV*} & 0 & -\beta_{k\lambda}^{VV*} \\ -\beta_{k\lambda}^{UU} & 0 & \alpha_{k\lambda}^{UU} & 0 \\ 0 & -\beta_{k\lambda}^{VV} & 0 & \alpha_{k\lambda}^{VV} \end{pmatrix}. \quad (40)$$

Explicitly, using the dictionary (33) now applied to Minkowski modes, the four coefficients  $S_{\omega k}^{1\nu}$  are

$$\begin{aligned} \alpha_{\omega k}^{UU} &= \delta(\omega - k) - i \int_0^\infty d\lambda (\alpha_{\omega\lambda}^{UU} T_\lambda^{UU} \alpha_{k\lambda}^{UU*} + \beta_{\omega\lambda}^{UU*} \bar{T}_\lambda^{UU*} \beta_{k\lambda}^{UU}), \\ \alpha_{\omega k}^{UV} &= -i \int_0^\infty d\lambda (\alpha_{\omega\lambda}^{UU} T_\lambda^{UV} \alpha_{k\lambda}^{VV*} + \beta_{\omega\lambda}^{UU*} \bar{T}_\lambda^{UV*} \beta_{k\lambda}^{VV}), \\ \beta_{\omega k}^{UU*} &= i \int_0^\infty d\lambda (\alpha_{\omega\lambda}^{UU} T_\lambda^{UU} \beta_{k\lambda}^{UU*} + \beta_{\omega\lambda}^{UU*} \bar{T}_\lambda^{UU*} \alpha_{k\lambda}^{UU}), \\ \beta_{\omega k}^{UV*} &= i \int_0^\infty d\lambda (\alpha_{\omega\lambda}^{UU} T_\lambda^{UV} \beta_{k\lambda}^{VV*} + \beta_{\omega\lambda}^{UU*} \bar{T}_\lambda^{UV*} \alpha_{k\lambda}^{VV}). \end{aligned} \quad (41)$$

Similar equations give the expressions for the remaining components of  $S_{\omega k}^{\mu\nu}$ . We have written  $\mathbf{S}_\lambda$  as  $\mathbf{S}_\lambda = \mathbf{1} - i \mathbf{T}_\lambda$  (and  $\bar{\mathbf{S}}_\lambda = \mathbf{1} - i \bar{\mathbf{T}}_\lambda$ ) in order to extract the trivial part of the diagonal elements. This trivial part leads to the delta function in the first equation. The usefulness of the writing is that it will be easily related to the perturbative expressions we shall encounter in the next Section.

Equations (41) are the central result of this Section. They give the *in-out* overlaps of Minkowski modes in terms of the matrices  $\mathbf{T}_\lambda, \bar{\mathbf{T}}_\lambda$  computed in the rest frame of the mirror and the overlaps between the free (unscattered) Minkowski and proper time modes.

It is then easy to obtain the mean flux emitted by this partially transmitting non-inertial mirror when the initial state of the field is the Minkowski vacuum. The same algebra which gave eq. (16) now gives

$$\langle T_{VV} \rangle = \langle T_{VV} \rangle^{particle} + \langle T_{VV} \rangle^{anti-particle}, \quad (42)$$

where

$$\begin{aligned} \langle T_{VV} \rangle^{particle} &= \Re \left[ \sum_{j=U,V} \iint_0^\infty d\omega d\omega' \frac{\sqrt{\omega\omega'}}{2\pi} \right. \\ &\quad \left. \left[ e^{-i(\omega'-\omega)V} \left( \int_0^\infty dk \beta_{\omega k}^{Vj*} \beta_{\omega'k}^{Vj} \right) - e^{-i(\omega'+\omega)V} \left( \int_0^\infty dk \bar{\alpha}_{\omega k}^{Vj*} \beta_{\omega'k}^{Vj} \right) \right] \right]. \end{aligned} \quad (43)$$

$\langle T_{VV} \rangle_{anti-particle}$  is given by the same expression with  $\bar{\alpha}, \beta$  replaced by  $\alpha, \bar{\beta}$ .  $\langle T_{VV} \rangle$  possesses the same structure as eq. (16). However, four kinds of coefficients  $\alpha, \beta$  should be considered since we are dealing with *partial* reflection of *charged* particles.

When the scattering is independent of the energy and the charge of the particles *i.e.* when  $R$  and  $\phi$  defined in eq. (29) are independent of  $\lambda$  and when  $\bar{\mathbf{S}}_\lambda^* = \mathbf{S}_{-\lambda}$ , the integration over  $\lambda$  can be trivially performed as it expresses the completeness of the  $\varphi_\lambda$  modes. In this case, as in the DF model, one has  $\beta_{\omega k}^{UU} = \beta_{\omega k}^{VV} = 0$ . One also finds that the emitted flux is simply

$$\langle T_{VV} \rangle = R^2 \langle T_{VV} \rangle_{DF} , \quad (44)$$

where  $\langle T_{VV} \rangle_{DF}$  is the flux found in the DF model, see (16).

Instead, when  $R$  and  $\phi$  depend on the energy and/of the charge,  $\beta_{\omega k}^{UU}$  and  $\beta_{\omega k}^{VV}$  will be, in general<sup>4</sup>, different from zero. In this case, one also loses the possibility of re-expressing the flux in terms of the derivatives of the trajectory as we did it in eq. (19). This can be understood from eqs. (41) : when expressing  $\mathbf{T}_\lambda$  as a series in powers of  $\lambda$ , one would obtain for  $\langle T_{VV} \rangle$  a non-local expression in  $V$  unless the series in  $\lambda$  stops after a finite number of terms.

### 2.3 Additional remarks

In this subsection, we relate the matrices  $\mathbf{S}_{\lambda\lambda'}$  and  $\mathbf{S}_{\omega k}$  which act linearly on *in* and *out* operators to the conventional  $S$  matrix acting on multi-particle states in the Fock space. With this identification we shall be able to relate the Bogoliubov coefficients eq. (41) to *transition amplitudes* and not only to expectation values as in eq. (43).

By definition[24], the action of this operator on states and operators is the following

$$\begin{aligned} |0_{in}\rangle &= \hat{S} |0_{out}\rangle, \\ a_\lambda^{i,out} &= \hat{S}^{-1} a_\lambda^{i,in} \hat{S} \quad , \quad b_\lambda^{i,out\dagger} = \hat{S}^{-1} b_\lambda^{i,in\dagger} \hat{S} . \end{aligned} \quad (45)$$

Since we are dealing with elastic scattering, this operator contains exactly the same information as the matrices  $\mathbf{S}_\lambda, \bar{\mathbf{S}}_\lambda$ . Indeed, the block diagonal character of eq. (30) and the linearity of eq. (31) tell us that  $\hat{S}$  is the exponential of a quadratic form of proper-time operators  $a_\lambda, b_\lambda$ :

$$\hat{S} = e^{-i(a_\lambda^{i,in} s_{\lambda\lambda'}^{ij} a_{\lambda'}^{j,in\dagger} - b_\lambda^{i,in\dagger} \bar{s}_{\lambda\lambda'}^{ij} b_{\lambda'}^{j,in})} . \quad (46)$$

Then straightforward algebra gives

$$\begin{aligned} \mathbf{s}_{\lambda\lambda'} &= \delta(\lambda - \lambda') \begin{pmatrix} \phi & \text{Arcsin}(R)e^{i\theta} \\ \text{Arcsin}(R)e^{-i\theta} & \phi \end{pmatrix} , \\ \bar{\mathbf{s}}_{\lambda\lambda'} &= \mathbf{s}_{\lambda\lambda'}(\bar{R}, \bar{\theta}, \bar{\phi}) , \end{aligned} \quad (47)$$

where  $R, \theta, \phi$  have been defined in eq. (29) and  $\bar{R}, \bar{\theta}, \bar{\phi}$  are defined in the same way from  $\bar{\mathbf{S}}_\lambda$ . We note that in the DF model, *i.e.* in the limit of perfect reflection,  $\mathbf{s}_{\lambda\lambda'}$  is given by

$$\mathbf{s}_{\lambda\lambda'}^{DF} = \delta(\lambda - \lambda') \left( \frac{\pi}{2} \right) \begin{pmatrix} 1 & e^{i\theta} \\ e^{-i\theta} & 1 \end{pmatrix} = \bar{\mathbf{s}}_{\lambda\lambda'}^{DF} . \quad (48)$$

Although the configurations on the left and on the right of the mirror completely decouple, the S-matrix  $\hat{S}$  treats both sides simultaneously.

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<sup>4</sup> $\beta_{\omega k}^{UU} = 0$  requires that  $\bar{T}_\lambda^{UU*} = -T_{-\lambda}^{UU}$  for all  $\lambda > 0$ , and similarly for the  $VV$  coefficients. In the next Section, we shall see that condition is satisfied for the time independent couplings with U(1) symmetry.

To anticipate the expression of  $\hat{S}$  in terms of Minkowski operators which will mix creation and destruction operators, it is convenient to rewrite eq. (46) in term of the 4-vector  $a_\lambda^{\mu,in}$ :

$$\hat{S} = e^{-i(a_\lambda^{\mu,in} s_{\lambda\lambda'}^{\mu\nu} a_{\lambda'}^{\nu,in\dagger})} \quad \text{with} \quad (s_{\lambda\lambda'}^{\mu\nu}) = \begin{pmatrix} s_{\lambda\lambda'}^{ij} & 0 \\ 0 & -\bar{s}_{\lambda\lambda'}^{ij} \end{pmatrix}. \quad (49)$$

To obtain the expression of  $\hat{S}$  in terms of the Minkowski operators  $a_k^{i,in}, b_k^{i,in}$ , it suffices to use the matrix  $\mathcal{B}_{\omega\lambda}$  to replace proper-time operators by Minkowski ones. Explicitly one obtains

$$\hat{S} = e^{-i(a_\omega^{\mu,in} s_{\omega\omega'}^{\mu\nu} a_{\omega'}^{\nu,in\dagger})} \quad \text{with} \quad s_{\omega\omega'}^{\mu\nu} = \mathcal{B}_{\omega\lambda}^{\mu\mu'} s_{\lambda\lambda'}^{\mu'\nu'} (\mathcal{B}_{\omega'\lambda'}^\dagger)^{\nu\nu'}. \quad (50)$$

Formally,  $\hat{S}$  provides the answer to all questions concerning asymptotic states and expectation values. For instance, the probability amplitude governing the (Minkowski) vacuum decay, eq. (109), is simply

$$Z^{-1} = \langle 0_{out} | 0_{in} \rangle = \langle 0_{in} | \hat{S} | 0_{in} \rangle. \quad (51)$$

Similarly, the probability amplitude for an initial quantum of momentum  $k$  to be scattered and for no pair to be created is

$$\langle 0_{out} | a_\omega^{i,out} a_k^{j,in\dagger} | 0_{in} \rangle = \langle 0_{in} | a_\omega^{i,in} \hat{S} a_k^{j,in\dagger} | 0_{in} \rangle = \frac{1}{Z} (\alpha^{-1})_{k\omega}^{ji}. \quad (52)$$

The last equality is easily obtained by using eq. (32) to express  $a_\omega^{i,out}$  in terms of  $a_k^{j,in}$  and  $b_\omega^{j,out\dagger}$ . In the same way, the Bogoliubov coefficient  $\beta$  is related to probability amplitude to find a pair of *out* quanta in the *in* vacuum by

$$\beta_{\omega k}^{ij} (\bar{\alpha}^{-1})_{k\omega'}^{ji'} = - \frac{\langle 0_{out} | a_\omega^{i,out} b_{\omega'}^{i',out} | 0_{in} \rangle}{\langle 0_{out} | 0_{in} \rangle} = - \frac{\langle 0_{in} | a_\omega^{i,in} b_{\omega'}^{i',in} \hat{S} | 0_{in} \rangle}{\langle 0_{in} | \hat{S} | 0_{in} \rangle}. \quad (53)$$

It should be stressed that these relations *determine* the physical interpretation of the overlaps  $\alpha, \beta$  given in eq. (41). In fact the second quantized framework was never used to obtain eq. (41): only the linearity of the relations and the orthonormal character of the proper time and the Minkowski modes basis were exploited.

The physical interpretation of  $\alpha, \beta$  is the following: to first order in the transfer matrix  $\mathbf{T}_\lambda$ ,  $\alpha$  ( $\beta$ ) divided by  $Z$  gives the probability amplitude to scatter a quantum (to produce a pair of quanta), since  $\alpha^{-1} \simeq 1 + iT$  ( $\beta \simeq -iT$ ). Upon considering higher order terms in  $\mathbf{T}_\lambda$ , one loses the simplicity of the relationship so as to get the above equations. The simple relation in the linear regime will be nicely confirmed in the next Section, when using perturbation theory. We shall see in particular that the division by  $Z$  corresponds to the usual restriction of keeping only the connected graphs engendered by the development of  $\hat{S} = T e^{-ig \int dt H}$  in powers of  $g$ . We shall further comment on these aspects at the end of Section 3.

### 3 The self-interacting model

In this section we introduce a model based on self-interactions which derives from an action principle[26, 27, 10]. In a first part we consider time independent couplings. In

this case, re-summing the Born series leads to diagonal matrices in the proper-time energy  $\lambda$  with parameters  $R$  and  $\phi$  which depend on  $\lambda$  according to the number of derivatives in the interaction hamiltonian. This model will be generalized by considering a thick mirror with a non-zero width. Using a perturbative approach, we shall see that the thickness acts as a UV cutoff.

In the second part, we work with time dependent couplings. We shall work perturbatively, up to the second order in the interactions. The novelty concerns the transients induced by the switching on and off of the coupling.

### 3.1 The scattering with $g$ constant

To exploit the fact that the coupling is  $\tau$  independent, it is convenient to work with the coordinates  $(\tau, \rho)$  in which the mirror is at rest. In these coordinates, the interaction Lagrangian reads :

$$L_{int} = g \iint_{-\infty}^{+\infty} d\tau d\rho f(\rho) J(\Phi(\tau, \rho), \Phi^\dagger(\tau, \rho)). \quad (54)$$

$g$  is the coupling parameter,  $f$  is a real function which specifies the thickness of the mirror and which is normalized as follows  $\int_{-\infty}^{+\infty} d\rho f(\rho) = 1$ .  $J$  is an hermitian operator which is quadratic in the complex field. We shall consider three different cases:  $\Phi^\dagger \Phi + \Phi \Phi^\dagger$ ,  $\Phi^\dagger \overleftrightarrow{\partial}_\tau \Phi$ , and  $\partial_\tau \Phi^\dagger \partial_\tau \Phi + \partial_\tau \Phi \partial_\tau \Phi^\dagger$ . In the following equations, we shall present the details only with the second expression. At the end of the derivation, we shall give the final results for the two other cases.

Given eq. (54), eq. (1) is now replaced by

$$(\partial_\tau^2 - \partial_\rho^2) \Phi(\tau, \rho) = g f(\rho) 2i \partial_\tau \Phi. \quad (55)$$

Being linear, the solution can be expressed as

$$\begin{aligned} \Phi(\tau, \rho) &= \Phi^{in}(\tau, \rho) + g \iint_{-\infty}^{+\infty} d\tau' d\rho' G^{ret}(\tau, \rho; \tau', \rho') f(\rho') 2i \partial_{\tau'} \Phi(\tau', \rho') \\ &= \Phi^{out}(\tau, \rho) + g \iint_{-\infty}^{+\infty} d\tau' d\rho' G^{adv}(\tau, \rho; \tau', \rho') f(\rho') 2i \partial_{\tau'} \Phi(\tau', \rho'), \end{aligned} \quad (56)$$

in terms of the homogeneous solution  $\Phi^{in}$  ( $\Phi^{out}$ ) which determines the initial (final) data. The retarded and advanced Green functions are defined, as usual, by

$$\begin{aligned} G^{ret}(\tau, \rho; \tau', \rho') &= \iint_{-\infty}^{+\infty} d\lambda dl \frac{1}{4\pi^2} \frac{e^{-i\lambda(\tau-\tau') + il(\rho-\rho')}}{l^2 - (\lambda + i\epsilon)^2} \quad (= 0 \text{ for } \tau' > \tau), \\ G^{adv}(\tau, \rho; \tau', \rho') &= \iint_{-\infty}^{+\infty} d\lambda dl \frac{1}{4\pi^2} \frac{e^{-i\lambda(\tau-\tau') + il(\rho-\rho')}}{l^2 - (\lambda - i\epsilon)^2} \quad (= 0 \text{ for } \tau' < \tau). \end{aligned} \quad (57)$$

To exploit the time independence of the coupling  $g$ , we work at fixed energy with

$$\varphi_\lambda(\rho) = \int_{-\infty}^{+\infty} d\tau \frac{1}{2\pi} \Phi(\tau, \rho) e^{i\lambda\tau}. \quad (58)$$

In Fourier transform, eqs. (56) give

$$\begin{aligned} \varphi_\lambda(\rho) &= \varphi_\lambda^{in}(\rho) + ig \int_{-\infty}^{+\infty} d\rho' f(\rho') \varphi_\lambda(\rho') e^{i\lambda|\rho-\rho'|} \\ &= \varphi_\lambda^{out}(\rho) - ig \int_{-\infty}^{+\infty} d\rho' f(\rho') \varphi_\lambda(\rho') e^{-i\lambda|\rho-\rho'|}. \end{aligned} \quad (59)$$

These equations have been obtained by using

$$\int_{-\infty}^{+\infty} dl \frac{e^{il(\rho-\rho')}}{l^2 - (\lambda \pm i\epsilon)^2} = \frac{\pm 2i\pi}{2(\lambda \pm i\epsilon)} e^{\pm i\lambda|\rho-\rho'|} . \quad (60)$$

We now decompose the quantized modes  $\varphi_\lambda^{in}$  in terms of creation and destruction operators,

$$\begin{aligned} \varphi_\lambda^{in}(\rho) &= \frac{1}{\sqrt{4\pi\lambda}} (a_\lambda^{U,in} e^{i\lambda\rho} + a_\lambda^{V,in} e^{-i\lambda\rho}) \quad \text{for } \lambda > 0 \\ &= \frac{1}{\sqrt{4\pi|\lambda|}} (b_{|\lambda|}^{U,in\dagger} e^{-i|\lambda|\rho} + b_{|\lambda|}^{V,in\dagger} e^{i|\lambda|\rho}) \quad \text{for } \lambda < 0. \end{aligned} \quad (61)$$

We do the same for the *out* modes. Then, for  $f(\rho) = \delta(\rho - \rho_0)$ , in the limit  $\epsilon \rightarrow 0$ , eqs. (59) give :

$$\begin{pmatrix} a_\lambda^{U,out} \\ a_\lambda^{V,out} \end{pmatrix} = \frac{1}{1-ig} \begin{pmatrix} 1 & ig e^{-2i\lambda\rho_0} \\ ig e^{2i\lambda\rho_0} & 1 \end{pmatrix} \begin{pmatrix} a_\lambda^{U,in} \\ a_\lambda^{V,in} \end{pmatrix} . \quad (62)$$

We recover the linear structure of  $\mathbf{S}_\lambda$  in eq. (25). Since the unitarity of  $\mathbf{S}_\lambda$  provides  $a_\lambda^{i,out} = S_\lambda^{ij*} a_\lambda^{j,in}$ , when using the definitions of eq. (29), we obtain

$$R = \frac{g}{\sqrt{1+g^2}}, \quad \phi = \text{Arctan}(g) \quad \text{and} \quad \theta = 2\lambda\rho_0 . \quad (63)$$

In the strong coupling limit (*i.e.* for  $g \rightarrow +\infty$ ) one obtains total reflexion (26) in a  $\lambda$  independent manner. This is a special feature of the coupling  $J = \Phi^\dagger i \overleftrightarrow{\partial}_\tau \Phi$  which is associated with a dimensionless  $g$ .

This analysis can be repeated with the two other operators previously defined. The presence or the absence of derivatives in  $J$  modifies the IR or UV behavior of  $R$ . For  $\Phi^\dagger \Phi + \Phi \Phi^\dagger$  one obtains [26] :

$$R_\lambda = \frac{g/\lambda}{\sqrt{1+g^2/\lambda^2}}, \quad \phi_\lambda = \text{Arctan}(g/\lambda) \quad \text{and} \quad \theta = 2\lambda\rho_0 . \quad (64)$$

In this case, the mirror is totally reflecting in the IR. This leads to strong IR divergences when considering time dependent coupling  $g$ . On the contrary, when using  $\partial_\tau \Phi^\dagger \partial_\tau \Phi + \partial_\tau \Phi \partial_\tau \Phi^\dagger$ , we get

$$R_\lambda = \frac{g\lambda}{\sqrt{1+g^2\lambda^2}}, \quad \phi_\lambda = \text{Arctan}(g\lambda) \quad \text{and} \quad \theta = 2\lambda\rho_0 . \quad (65)$$

In this case, the mirror is transparent in the IR limit. This useful property will be exploited in Section 4.

One notices that the transfer matrix can be expressed in a general way according to the number  $n$  of derivatives  $\partial_\tau$  in the interaction term:

$$\mathbf{T}_\lambda = \frac{-g\lambda^{n-1}}{1-ig\lambda^{n-1}A_\epsilon} \begin{pmatrix} 1 & e^{2i\lambda\rho_0} \\ e^{-2i\lambda\rho_0} & 1 \end{pmatrix} . \quad (66)$$



In this expression, we have not taken the limit  $\epsilon \rightarrow 0$ . The function  $A_\epsilon = \lambda/(\lambda + i\epsilon)$  determines the analytical properties of  $\mathbf{T}_\lambda$  in the complex  $\lambda$  plane. The specification of the pole of  $A_\epsilon$  follows from that of  $G^{ret}$  in eq. (57). It guarantees that causality will be respected[26]. This crucial ingredient was missing in Section 2.2 wherein the matrix  $\mathbf{T}_\lambda$  can be chosen from the outset. In that kinematic framework, the analytical properties should be imposed by hand if one wishes to implement causality. On the contrary, in the present case causality follows from the Heisenberg equations.

Eqs. (59) and (61) also determine the relation between the anti-particle *in* and *out* operators  $b_\lambda^{i\dagger}$ . By direct computation one finds  $\bar{\mathbf{T}}_\lambda^* = -\mathbf{T}_{-\lambda}$ . This is precisely the condition which gives  $\beta_{\omega k}^{UU} = \beta_{\omega k}^{VV} = 0$ , see the former footnote. When using  $\mathbf{T}_\lambda, \bar{\mathbf{T}}_\lambda$  in eqs. (41), we obtain the Bogoliubov coefficients relating inertial modes. And from these coefficients, one gets the mean value of energy flux  $T_{VV}$  as in eq. (43), but with causality built in.

We now study the case of a thick mirror with  $J$  given by  $\Phi^\dagger i \overleftrightarrow{\partial}_\tau \Phi$ . To display the effects of  $f(\rho)$ , it is convenient to work with the (spatial) Fourier components. Eqs. (59) become

$$\begin{aligned} \varphi_{\lambda,l} &= \varphi_{\lambda,l}^{in} - \frac{2g\lambda}{(\lambda + i\epsilon)^2 - l^2} \int_{-\infty}^{+\infty} dl' f_{l-l'} \varphi_{\lambda,l'} \\ &= \varphi_{\lambda,l}^{out} - \frac{2g\lambda}{(\lambda - i\epsilon)^2 - l^2} \int_{-\infty}^{+\infty} dl' f_{l-l'} \varphi_{\lambda,l'} . \end{aligned} \quad (67)$$

For an arbitrary window function  $f$ , these equations do not lead to analytic relations between asymptotic *in* and *out* fields. Therefore, we use perturbation theory. To first order in  $g$  we get

$$\mathbf{T}_\lambda^f = -g \begin{pmatrix} 1 & 2\pi f_{2\lambda} \\ 2\pi f_{2\lambda}^* & 1 \end{pmatrix} \quad (68)$$

For a normalized gaussian function  $f$  centered on  $\rho_0$ , the non-diagonal terms which determine the reflection probability are  $ge^{\pm 2i\lambda\rho_0} e^{-2\lambda^2\sigma^2}$ . Therefore,  $\sigma$ , the spread of the mirror reduces the reflection of high frequencies: for  $\lambda \gg 1/\sigma$  the mirror is completely transparent (this is also true for the two other  $J$ 's).

## 3.2 The scattering with $g$ time dependent

In this subsection, the coupling parameter is a function of the proper time  $g(\tau) = g f(\tau)$  where  $f(\tau)$  is normalized by  $\int_{-\infty}^{+\infty} d\tau f(\tau) = 2T$  with  $2T$  the proper-time lapse during which the interactions are turned on. Unlike what we had in the former subsection, resumming the Born series is no longer possible since the time dependence of the coupling destroys the decoupling of the equations into sectors at fixed frequency  $\lambda$ . In fact we meet a situation analogous of that of a thick mirror which mixed different momenta. Thus, we shall work perturbatively: all quantities will be evaluated up to the second order in  $g$ .

We remind the reader that in the interacting picture, the operator  $\Phi$  evolves freely, *i.e.* with  $g = 0$ . Therefore the *in* operators  $a_\omega, b_\omega$  coincide with the *out* operators and are equal to the usual Minkowski operators. Hence, they define the (Minkowski) vacuum  $|0\rangle$ . Instead, the states evolve through the action of the time ordered operator:

$$|\Psi(t = +\infty)\rangle = T e^{iL} |\Psi(t = -\infty)\rangle, \quad (69)$$

where  $L = g \int d\tau f(\tau) J$ . Since the trajectory is timelike the time ordering with respect to the Minkowski time  $t$  is equivalent to that of the proper-time  $\tau$ .

To make contact with Section 2, we shall work in this section with the state  $|\Psi_0(t)\rangle$  which is equal to  $|0\rangle$  for  $t = \tau = -\infty$ . When expressing its final value in the basis of the unperturbed states, *i.e.* the states which would have been stationary in the absence of interactions, we get

$$|\Psi_0(\tau = +\infty)\rangle = |0\rangle + \sum_{i,j} \iint_0^\infty d\omega d\omega' (B_{\omega\omega'}^{ij} + C_{\omega\omega'}^{ij}) |\omega\omega'\rangle_{ij} \quad (70)$$

where

$$B_{\omega\omega'}^{ij} = ig \langle 0 | a_\omega^i b_{\omega'}^j \left( \int_{-\infty}^{+\infty} d\tau f(\tau) J(\tau) \right) | 0 \rangle, \quad |\omega\omega'\rangle_{ij} \equiv a_\omega^{i\dagger} b_{\omega'}^{j\dagger} | 0 \rangle, \quad (71)$$

$$C_{\omega\omega'}^{ij} = -g^2 \langle 0 | a_\omega^i b_{\omega'}^j \left( \int_{-\infty}^{+\infty} d\tau \int_{-\infty}^\tau d\tau' f(\tau) f(\tau') J(\tau) J(\tau') \right) | 0 \rangle_c. \quad (72)$$

We have limited the expansion in  $g$  to these three terms since we shall compute the energy-momentum tensor up to  $g^2$  terms only. As before,  $i, j$  denote the  $U, V$  sectors and  $\omega, \omega'$  Minkowski energies. The symbol  $\langle \rangle_c$  means that only the *connected* part of the expectation value is kept. This restriction follows from the fact that the contribution of the disconnected graphs cancels out since they also appear in the denominator of the expectation values, see *e.g.* [24].

Using eq. (70), the expectation value of  $T_{VV}$  is given by :

$$\begin{aligned} \langle T_{VV} \rangle &= \langle \Psi_0(\tau = +\infty) | T_{VV} | \Psi_0(\tau = +\infty) \rangle_c \\ &= \Re \left[ \sum_j \iint_0^\infty d\omega d\omega' \frac{\sqrt{\omega\omega'}}{2\pi} \left[ e^{-iV(\omega' - \omega)} \int_0^\infty dk \left( B_{\omega k}^{Vj*} B_{\omega' k}^{Vj} + \bar{B}_{\omega k}^{Vj*} \bar{B}_{\omega' k}^{Vj} \right) \right. \right. \\ &\quad \left. \left. - e^{-iV(\omega' + \omega)} \left( B_{\omega\omega'}^{VV} + \bar{B}_{\omega\omega'}^{VV} + C_{\omega\omega'}^{VV} + \bar{C}_{\omega\omega'}^{VV} \right) \right] \right], \end{aligned} \quad (73)$$

where  $\bar{B}_{\omega k}^{ij}$  and  $\bar{C}_{\omega\omega'}^{VV}$  are related to the unbar quantities by inverting particle and anti-particle operators, thus  $\bar{B}_{\omega\omega'}^{ij} = B_{\omega'\omega}^{ji}$  and  $\bar{C}_{\omega\omega'}^{VV} = C_{\omega'\omega}^{VV}$ .

Since the integral of the second term in eq. (73) vanishes and since bar and unbar quantities differ at most by a phase, the total energy received on the  $V$  part of  $\mathcal{J}^+$  is :

$$\langle H_V \rangle \equiv \int_{-\infty}^{+\infty} dV \langle T_{VV} \rangle_c = 2 \sum_j \int_0^\infty d\omega \omega \int_0^\infty dk |B_{\omega k}^{Vj}|^2. \quad (74)$$

Hence only the  $B$  terms contribute to the energy as the  $\beta$  terms did in eq. (17).

In order to compute the local properties of the flux, we need to compute the second term of eq. (73). To this end we decompose  $C_{\omega\omega'}^{VV}$  into two parts :

$$C_{\omega\omega'}^{VV} = R_{\omega\omega'}^{VV} - \langle 0 | a_\omega^V b_{\omega'}^V \mathcal{D} | 0 \rangle_c, \quad (75)$$

where

$$R_{\omega\omega'}^{VV} = -\frac{1}{2} \langle 0 | a_\omega^V b_{\omega'}^V L L | 0 \rangle_c, \quad (76)$$

$$\mathcal{D} = \frac{g^2}{2} \left( \int_{-\infty}^{+\infty} d\tau \int_{-\infty}^{+\infty} d\tau' f(\tau) f(\tau') \epsilon(\tau - \tau') J(\tau) J(\tau') \right) \quad (77)$$

and  $\epsilon(\tau - \tau') = \theta(\tau - \tau') - \theta(\tau' - \tau)$ . Then,  $\langle T_{VV} \rangle_{\mathcal{D}}$ , the contribution of  $\mathcal{D}$  to  $\langle T_{VV} \rangle$ , enjoys the following properties. First it carries no energy. This is obvious since it is built with terms which all contain  $e^{iV(\omega+\omega')}$ . Secondly, it vanishes for  $f(\tau) = cst$ . This can be understood from the fact that the time ordering properties can be encoded in the analytical properties of the matrix  $\mathbf{T}_\lambda$  which is diagonal in  $\lambda$ , see eq. (66). This means that this term modifies the shape of the transients related to the switching on and off of the interaction but without affecting their energy content. In the rest of the paper, we shall therefore ignore this term.

We now compute  $R_{\omega\omega'}^{VV}$ . Since only the connected part should be kept, we can insert the following operator between the two operators  $L$  in eq. (76)

$$\sum_{i,j} \int_0^\infty dk \int_0^\infty dk' a_k^{i\dagger} b_{k'}^{j\dagger} |0\rangle \langle 0| a_k^i b_{k'}^j. \quad (78)$$

Grouping together, as in eq. (73), the first order and the second order contribution in  $g$ , we get

$$B_{\omega\omega'}^{VV} + \bar{B}_{\omega\omega'}^{VV} + R_{\omega\omega'}^{VV} + \bar{R}_{\omega\omega'}^{VV} = \sum_i \int_0^\infty dk (\bar{A}_{\omega k}^{Vj*} B_{\omega'k}^{Vj} + A_{\omega'k}^{Vj*} \bar{B}_{\omega k}^{Vj}) \quad (79)$$

with

$$A_{\omega k}^{ij*} = \langle 0| a_\omega^i (1 + iL) a_k^{j\dagger} |0\rangle_c \quad \text{and} \quad \bar{A}_{\omega k}^{ij*} = \langle 0| b_\omega^i (1 + iL) b_k^{j\dagger} |0\rangle_c. \quad (80)$$

Hence we find that  $\langle T_{VV} \rangle$  is given by eq. (42) with

$$\begin{aligned} \langle T_{VV} \rangle^{particle} &= \Re \sum_j \int_0^\infty d\omega \int_0^\infty d\omega' \frac{\sqrt{\omega\omega'}}{2\pi} \\ &\quad \left[ e^{-iV(\omega'-\omega)} \left( \int_0^\infty dk B_{\omega k}^{Vj*} B_{\omega'k}^{Vj} \right) - e^{-iV(\omega'+\omega)} \left( \int_0^\infty dk \bar{A}_{\omega k}^{Vj*} B_{\omega'k}^{Vj} \right) \right]. \end{aligned} \quad (81)$$

$\langle T_{VV} \rangle_{anti-particle}$  is given by the same expression with  $\bar{A}_{\omega k}^{Vj}, B_{\omega k}^{Vj}$  replaced by  $A_{\omega k}^{Vj}, \bar{B}_{\omega k}^{Vj}$ .

Thus, to second order in  $g$ , we recover the structure of eq. (43) which gives the flux emitted by a partially transmitting mirror. The Bogoliubov coefficients  $\alpha_{\omega k}^{Vj}$  and  $\beta_{\omega k}^{Vj}$  have been replaced by the transition amplitudes  $A_{\omega k}^{Vj}$  and  $B_{\omega k}^{Vj}$ . In this we recover the correspondence of eqs. (52) and (53) when considered to first order in the transfer matrix  $\mathbf{T}_\lambda$ . This is not surprising since the evolution operator  $T e^{iL}$  which defines  $A_{\omega k}^*$  and  $B_{\omega k}$ , given in eqs. (80) and (71), is the operator  $\hat{S}$  of eq. (45).

This correspondence is nicely illustrated in the case where  $g(\tau) = g$  and  $J = \Phi^\dagger i \overleftrightarrow{\partial}_\tau \Phi$ . In this case, to order  $g$  but whatever is the mirror's trajectory  $U = U_d(V)$ , one has the following identities

$$A_{\omega k}^{VU} = g \alpha_{\omega k}, \quad B_{\omega k}^{VU} = g \beta_{\omega k}, \quad (82)$$

where  $\alpha_{\omega k}$  and  $\beta_{\omega k}$  are the Bogoliubov computed in the DF model. These relations establish that  $\alpha_{\omega k}$  and  $\beta_{\omega k}$  should be understood as transition amplitudes. This implies in particular that the momentum transfers to mirror (which have been neglected so far) are respectively  $\hbar(k+\omega)$  and  $\hbar(-k+\omega)$ . This imposes limitations when considering ultra-high (trans-Planckian) frequencies since neglecting the momentum transfers requires  $\hbar\omega \ll M$  where  $M$  is the mass of the mirror[10]. Thus, when high frequency quanta are emitted, the validity of the predictions obtained with a recoil-less model *must* be questioned[11].

## 4 Applications

First, we analyze the properties of the transients associated with the switching on and off when the mirror is at rest ( $z = 0$ ) and in Minkowski vacuum. Secondly, we generalize this analysis by replacing the Minkowski vacuum by a thermal bath. Then, we use the well-known parallel between inertial systems in a thermal bath and uniformly accelerated systems in vacuum to obtain a regularized expression of the flux emitted by a uniformly accelerated mirror.

### 4.1 The transients in vacuum

We first focus on the frequency content of the transients. For an inertial mirror at rest at  $z = 0$  in Minkowski vacuum, the transition amplitudes  $A$  and  $B$  of eqs. (80) and (71) can be expressed in terms of the Fourier transforms of  $f(t)$

$$f_\omega = \frac{1}{2\pi} \int dt f(t) e^{i\omega t} . \quad (83)$$

To order  $g$  we obtain

$$A_{\omega k}^{ij*} = \delta(\omega - k) \delta^{ij} + ig f_{\omega-k} \frac{j(\omega, k)}{\sqrt{\omega k}} , \quad (84)$$

$$B_{\omega k}^{ij} = ig f_{\omega+k} \frac{j(\omega, -k)}{\sqrt{\omega k}} , \quad (85)$$

where

$$j(\omega, k) = \begin{cases} 1 & \text{for } \Phi^\dagger \Phi + \Phi \Phi^\dagger \\ \omega + k & \text{for } \Phi^\dagger i \overleftrightarrow{\partial}_t \Phi \\ \omega k & \text{for } \partial_t \Phi^\dagger \partial_t \Phi + \partial_t \Phi \partial_t \Phi^\dagger \end{cases} . \quad (86)$$

Thus to order  $g^2$ , the mean number of  $V$  particles of energy  $\omega$  is given by :

$$\langle N_\omega^V \rangle = 2 \sum_j \int_0^\infty dk |B_{\omega k}|^2 . \quad (87)$$

The factor of 2 arises from the fact that the pair production amplitudes  $B$  are independent of the (relative) sign of the momentum of the two quanta.

Our aim is to describe the transients associated with the switching on and off of the coupling to the mirror. To this end and to be concrete, we shall work with the function

$$f(t) = \frac{1}{2} \left( \tanh\left(\frac{t+T}{\Delta}\right) - \tanh\left(\frac{t-T}{\Delta}\right) \right) . \quad (88)$$

It is constant during a lapse of time  $2T$  centered about  $t = 0$  and the time intervals of the switching on and off are  $\simeq 4\Delta$ . In the limit  $\Delta \rightarrow 0$ ,  $f$  tends to the square window  $[\theta(t+T) - \theta(t-T)]/2$ . The Fourier components of  $f$  are

$$f_\omega = \frac{\Delta}{2} \frac{\sin(\omega T)}{\sinh(\omega \pi \Delta / 2)} . \quad (89)$$

One sees that the UV behavior is exponentially damped by  $\Delta$ . On the contrary, in the IR,  $f_\omega \rightarrow T/\pi$  since the coupling lasts during  $2T$ .

When considering the first two cases of  $j(\omega, k)$  of eq. (86), these simple observations imply that the mean number  $\langle N_\omega \rangle$  is ill defined since the integral over  $k$  in eq. (87) diverges in the IR. Therefore to obtain well defined expressions we shall work with the third case. In this case one has

$$\langle N_\omega^V \rangle = \frac{g^2 \Delta^2}{2} \omega \int_0^\infty d\omega' \omega' \frac{\sin^2((\omega + \omega')T)}{\sinh^2((\omega + \omega')\pi\Delta/2)} . \quad (90)$$

It is perhaps appropriate to discuss the condition on the (dimensionfull) coupling constant  $g$  which guarantees the validity of a perturbative treatment limited to order  $g^2$ . The condition is that the mean number of quanta per quantum cell (which is equal to  $\langle N_\omega \rangle d\omega \simeq \langle N_\omega \rangle \pi/T$  in the limit  $\omega T \gg 1$ ) be well approximated by eq. (87). This requires that the probability to obtain two quanta in a cell is negligible with respect to that to obtain one. This translates mathematically by  $g^2 \ll T \Delta$  in the limit of interest  $T/\Delta \gg 1$ , i.e. when the flat plateau is much longer than the slopes. The condition  $g^2 \ll T \Delta$  means that the limit  $T \rightarrow \infty$  can be safely taken. Instead the limit  $\Delta \rightarrow 0$  is more delicate. A sufficient condition consists in working at fixed  $\tilde{g}^2 \ll 1$  where  $\tilde{g} = g(T\Delta)^{-1/2}$ . A stronger condition is to impose that the total number of particles emitted,  $\int_0^\infty \langle N_\omega^V \rangle d\omega$ , is finite in the limit  $\Delta \rightarrow 0$ . In this case,  $\bar{g} = g/\Delta$  should be held fixed.

When studying eq. (90), one first notices that in the limit  $T \rightarrow \infty$  with  $g$  and  $\Delta$  fixed the total number of particles emitted is independent of  $T$ , thereby not giving rise to a Golden Rule behavior characterized by a linear growth in  $T$ . Secondly,  $\langle N_\omega^V \rangle$  is maximum for  $\omega \propto 1/\Delta$ . Finally, for  $\omega\Delta \gg 1$  one has  $\langle N_\omega^V \rangle \simeq e^{-\pi\omega\Delta}$ . We thus find all the expected attributes of transients: their particle content is independent of the duration  $T$  when  $T/\Delta \gg 1$ , and their Fourier content is peaked around the adiabatic switching rate  $\Delta^{-1}$ .

We now study the space time repartition of the energy fluxes associated with these transients effects. We first notice that once the  $\mathcal{D}$ -term defined in eq. (75) as been dropped, the mean flux can be expressed as :

$$\langle T_{VV} \rangle = -2\Im m (\langle 0|T_{VV}L|0 \rangle) + \Re e (\langle 0|L [T_{VV}, L] |0 \rangle) . \quad (91)$$

Of course, by decomposing  $L$  and  $T_{VV}$  in terms of creation and annihilation operators, one would recover respectively the linear and the quadratic contribution of eq. (81). However, being interested in the space time properties, we shall not do so and shall work instead in the time ‘representation’ with the  $V$  part of the (positive frequency) Wightman function. It obeys

$$\partial_V W(V - V') = \partial_V \langle 0|\Phi^\dagger(V, U) \Phi(V', U')|0 \rangle = -\frac{1}{4\pi} \frac{1}{V - V' - i\epsilon} . \quad (92)$$

Using this function, the linear contribution in  $g$  reads

$$\begin{aligned} \langle T_{VV} \rangle_{lin} &= -8g \Im m \left[ \int dt f(t) \{ \partial_t \partial_V W(V - t) \}^2 \right] \\ &= \frac{g}{12\pi} \partial_t^3 f(t = V) . \end{aligned} \quad (93)$$

To obtain this result, we have integrated by part three times. The boundary contributions all vanish since  $f$  given in eq. (88) decreases faster than any power of  $t$ . The last integration

is trivially performed by using  $\Im m((x - i\epsilon)^{-1}) = \pi \delta(x)$ . These properties explain the local character of the expectation value<sup>5</sup>.

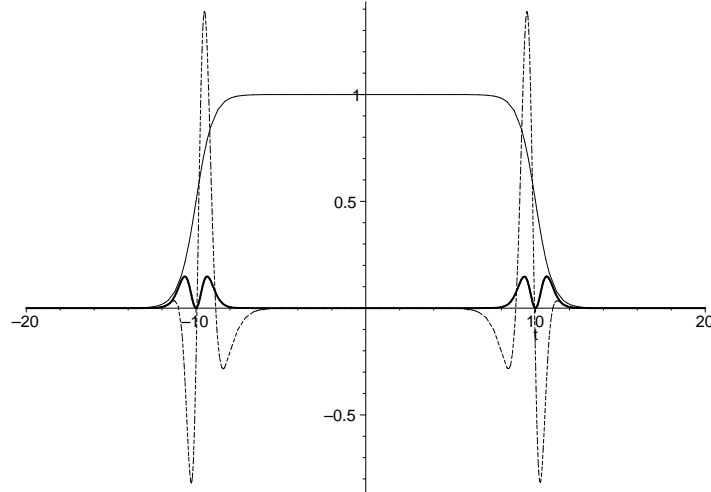
To evaluate the quadratic contribution in  $g$  we proceed along the same lines. We first evaluate the commutator so as to obtain a quadratic form in  $\Phi$  and  $\Phi_V$ , where  $\Phi_V$  means only the  $V$  part of the field operator  $\Phi$  should be kept. We notice that the derivatives  $\partial_t$  in  $J$  might be expressed as  $\partial_V$  since they are evaluated at  $z = 0$  but they act both on the  $V$  and the  $U$  part of  $\Phi$ . Using this notation, one finds

$$\begin{aligned} [T_{VV}, L] = & ig f(V) \left[ (\partial_V \Phi_V^\dagger \partial_V^2 \Phi + \partial_V \Phi_V \partial_V^2 \Phi^\dagger) + (\text{h.c.}) \right] \\ & + ig f'(V) \left[ (\partial_V \Phi_V^\dagger \partial_V \Phi + \partial_V \Phi_V \partial_V \Phi^\dagger) + (\text{h.c.}) \right]. \end{aligned} \quad (94)$$

Then the  $g^2$  contribution of  $T_{VV}$  is

$$\begin{aligned} \langle T_{VV} \rangle_{quadr} = & 16g^2 f(V) \Re \left( i \int dV' f(V') (\partial_{V'} \partial_V^2 W(V' - V)) (\partial_{V'} \partial_V W(V' - V)) \right) \\ & + 16g^2 \partial_V f \Re \left( i \int dV' f(V') (\partial_{V'} \partial_V W(V' - V))^2 \right) \\ = & -\frac{g^2}{12\pi} (f \partial_V^4 f + 2 \partial_V f \partial_V^3 f). \end{aligned} \quad (95)$$

Since neither  $f$  appears in eq. (93) nor  $f^2$  in eq. (95), one recovers the fact that an inertial mirror doesn't radiate while its coupling is constant. This is illustrated in Fig. 1.



**Figure 1:** The thin solid line is  $f(t)$  given by eq. (88), for  $T = 10$  and  $\Delta = 1$ . The dashed line is  $\langle T_{VV} \rangle_{quadr}$  and the thick line is the part of  $\langle T_{VV} \rangle$  which contributes to the energy, see eq. (97). These two curves have been plotted in the same arbitrary units. The behavior of  $\langle T_{VV} \rangle_{lin}$  is similar to  $\langle T_{VV} \rangle_{quadr}$ .

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<sup>5</sup> It should be pointed out that we could have written  $\langle T_{VV} \rangle_{lin}$  as a commutator. This however is not appropriate since one loses the analytical properties of  $W$  which are encoded by  $i\epsilon$  (They arise from frequency content of the vacuum and play a crucial role in defining the above expressions). By performing first the commutator (or equivalently by first taking the imaginary part in eq. (93)) one would obtain an ill defined expression. The same remark applies to the quadratic term in  $g$ . To obtain well defined expressions, only one commutation (and not two) should be done.

As in eq. (19), one can decompose  $\langle T_{VV} \rangle$  into two parts : a positive definite term and a total derivative which does not contribute to the total energy

$$\begin{aligned} \langle T_{VV} \rangle &= \langle T_{VV} \rangle_{lin} + \langle T_{VV} \rangle_{quadr} \\ &= \frac{g^2}{12\pi} (\partial_V^2 f)^2 - \frac{1}{12\pi} \partial_V \left[ -g \partial_V^2 f + g^2 \left( \frac{1}{2} \partial_V^4 (f^2) - \partial_V^2 ((\partial_V f)^2) \right) \right]. \end{aligned} \quad (96)$$

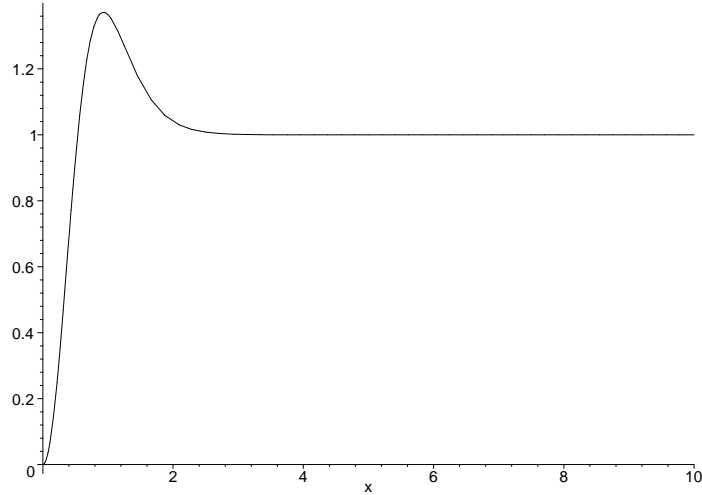
Thus the total energy is

$$\langle H_V \rangle = \frac{g^2}{12\pi} \int_{-\infty}^{+\infty} dV (\partial_V^2 f)^2 = \int_0^\infty d\omega \omega \langle N_\omega^V \rangle. \quad (97)$$

$\langle H_V \rangle$  is finite when the mean number  $\langle N_\omega \rangle$  decreases faster than  $\omega^{-2}$ . This is the case when working with eq. (88) at fixed  $\Delta \neq 0$ . In this case, one finds

$$\langle H_V \rangle = \frac{2g^2}{45\pi} \frac{1}{\Delta^3} F_1\left(\frac{T}{\Delta}\right). \quad (98)$$

The main feature  $\langle H_V \rangle$  is that it is independent of  $T$  in the limit  $T/\Delta \gg 1$ , see Fig. 2.



**Figure 2:** The plot  $F_1(x)$  defined in eq. (98) in such a way that  $F \rightarrow 1$  for  $x \rightarrow \infty$  where  $x = T/\Delta$ .

In conclusion, we consider the limit  $\Delta \rightarrow 0$  which corresponds to the situation studied in [20] and in [7] in view of its analogies with the residual flux emitted at the end of the evaporation of a black hole. In this limit,  $f(t)$  becomes a step function, the energy flux is concentrated in a narrow lapse  $\Delta$  and its frequency content diverges. In fact  $\langle T_{VV} \rangle$  becomes a distribution since it is built on the derivatives of  $f(t)$ . However the singularity is worse than a delta, as clearly seen from eq. (95). This means that the total energy emitted is also singular, as indicated in eq. (98). Moreover, there is no reason for the nature of the singularity to be universal. It depends on the number of derivatives in the Hamiltonian and it might also vary when considering higher orders in  $g$ . Hence the question: what is the flux emitted by the disappearance of the reflection condition, is not univocally defined. To have a well-defined question, one should first choose a regular model such as that defined by eq. (54) with  $J$  containing at least two derivatives and  $g(t)$  given by eq. (88), and only then consider the singular limit.

## 4.2 Flux and energy in a thermal bath

Our aim is to obtain a regularized expression for the flux emitted by a uniformly accelerated mirror. To this end, we shall use the isomorphism between the flux emitted by a mirror at rest in a heat bath at temperature  $a/2\pi$  and the Rindler flux emitted by a uniformly accelerated mirror of acceleration  $a$ .

In a thermal bath, the  $V$  part of the Wightman function obeys

$$\partial_V W^\beta(V - V') = -\frac{1}{4\pi} \frac{\pi}{\beta} \coth\left(\frac{\pi}{\beta}(V - V' - i\epsilon)\right). \quad (99)$$

It reduces to  $\partial_V W$  of eq. (92) in the zero temperature limit, i.e. for  $\beta \rightarrow \infty$ . When replacing  $W$  by  $W^\beta$  in eqs. (93) and (95) we obtain the mean flux emitted in a thermal bath. It can be shown to be<sup>6</sup>

$$\begin{aligned} \langle T_{VV} \rangle^\beta &= \frac{g}{12\pi} \partial_V^3 f - \frac{g^2}{12\pi} \left( f(V) \partial_V^4 f + 2\partial_V f \partial_V^3 f \right) \\ &\quad - \left( \frac{2\pi}{\beta} \right)^2 \left[ \frac{g}{12\pi} \partial_V f - \frac{g^2}{12\pi} \left( f(V) \partial_V^2 f + 2\partial_V f \partial_V f \right) \right]. \end{aligned} \quad (100)$$

The first two terms are equal to eq. (96) and the last two scale like  $(\Delta/\beta)^2$ . Thus, they are negligible in a low temperature limit,  $\beta \gg \Delta$ , and dominant in the high temperature regime.

We are now in position to obtain a regular expression for the flux emitted by a uniformly accelerated mirror in Minkowski vacuum. Using the well-known isomorphism between systems at rest in a thermal bath and accelerated systems in vacuum, the mean flux of Rindler energy emitted by a mirror of acceleration  $a$  is

$$\langle T_{vv}(v) \rangle^{acc} = \langle T_{VV}(V = v) \rangle^{\beta=2\pi/a}, \quad (101)$$

where  $v$  is the null advanced Rindler time ( $av = \ln(aV)$ ) when the mirror is located in the Right Rindler quadrant, ( $z < |t|$ ). The coupling between the mirror and the field is turned on during a proper time lapse  $2T$  and the switching on and off rate  $\Delta^{-1}$  is also measured with the proper time.

In the limit  $T \gg \beta$  and  $a^{-1}$ ,  $\langle T_{vv}(v) \rangle^{acc} \rightarrow 0$  at fixed  $|v| < T$  since the flux is localized in the transients of ‘thickness’  $\Delta$ . In this we recover the well known result that a uniformly accelerated mirror does not radiate. In the DF model, this immediately follows from eq. (19). (Notice that this vanishing is a universal property of accelerated systems when they have reached equilibrium with the Rindler bath[28, 29, 8, 16].) However this vanishing flux is accompanied by transients effects whose Minkowski properties become singular in the limit  $T \rightarrow \infty$  whatever is the value of  $\Delta$ . To have regular Minkowski properties means that the total energy

$$\langle H_V^{acc} \rangle = \int_{-\infty}^{+\infty} dv e^{av} \langle T_{vv}^{acc} \rangle, \quad (102)$$

and the mean number of Minkowski quanta emitted by the mirror be finite.

When requiring that  $\langle H_V^{acc} \rangle$  is finite, the coupling  $f(\tau)$  must decrease faster than the Doppler effect  $dV/dv = e^{av}$ . Using eq. (88), this implies  $a\Delta < 2$ . (A similar condition

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<sup>6</sup>The details of the calculation will be presented in [22].



also applies when considering the fluxes emitted by an accelerated two level atom[16].) Using eqs. (100), (101) and (102), we get

$$\langle H_V^{acc} \rangle = \frac{g^2}{12\pi} \int_{-\infty}^{+\infty} dv e^{av} \left[ (\partial_v^2 f)^2 + 2a^2 (\partial_v f)^2 \right]. \quad (103)$$

What we have learned from the condition  $a\Delta < 2$  is that the Rindler fluxes which lead to finite Minkowski energies are dominated by the vacuum effects governed by  $\Delta$  rather than by the temperature effects induced by the acceleration.

## A The *in* – *out* overlap in the non-stationary case

In order to have simple expressions for this overlap, we will use a discretized basis of wave packets in which the integrals are replaced by sums and Dirac distributions by Kronecker symbols.

Instead of working with the *in* and *out* basis, it is appropriate to define a third class of operators  $\hat{a}_\omega, \hat{b}_\omega$ . This new basis generalizes the Unruh modes[25, 8] since  $\hat{a}_\omega$  ( $\hat{b}_\omega$ ) is made out of  $a_k^{in}$  ( $b_k^{in}$ ) but is characterized by a fixed *out* frequency  $\omega$ :

$$\alpha_\omega \hat{a}_\omega = \sum_k \alpha_{\omega k}^* a_k^{in}, \quad \alpha_\omega \hat{b}_\omega = \sum_k \alpha_{\omega k}^* b_k^{in}. \quad (104)$$

The real coefficients  $\alpha_\omega$  are such that  $[\hat{a}_\omega, \hat{a}_\omega^\dagger] = 1$ , therefore  $\alpha_\omega^2 = \sum_k |\alpha_{\omega k}|^2$ . The notion of particle/anti-particle is obviously maintained since the  $\hat{a}$  are made of  $a^{in}$  only. Notwithstanding, for arbitrary  $\alpha_{\omega k}$  and  $\beta_{\omega k}$  this new basis is not orthogonal and the commutation rules are given by

$$[\hat{a}_\omega, \hat{a}_{\omega'}^\dagger] \equiv F_{\omega\omega'} = [\hat{b}_\omega, \hat{b}_{\omega'}^\dagger] = \frac{\sum_k \alpha_{\omega k}^* \alpha_{\omega' k}}{\alpha_\omega \alpha_{\omega'}}. \quad (105)$$

By construction from eq. (13), these new operators are related the *out* operators by

$$\begin{cases} a_\omega^{out} = \alpha_\omega \hat{a}_\omega - \sum_{\omega'} \alpha_{\omega\omega'} B_{\omega\omega'} \hat{b}_{\omega'}^\dagger \\ b_\omega^{out} = \alpha_\omega \hat{b}_\omega - \sum_{\omega'} \alpha_{\omega\omega'} B_{\omega\omega'} \hat{a}_{\omega'}^\dagger \end{cases} \quad \text{with } B_{\omega\omega'} \equiv \sum_k \beta_{\omega k} \alpha_{k\omega'}^{-1}, \quad (106)$$

where  $\alpha_{k\omega}^{-1}$  is the inverse matrix of  $\alpha_{\omega k}$ . ( $\alpha_{\omega k}$  is always invertible since otherwise there would exist incoming particles whose scattering would give only anti-particles.)

As for the Unruh modes, the new basis is useful to relate in a simple way the *out* vacuum to the *in* vacuum. Straightforward algebra indeed gives

$$|0_{out}\rangle = \frac{1}{Z} \exp \left( \sum_{\omega\omega'k} \frac{\alpha_{\omega'}}{\alpha_k} F_{\omega k}^{-1} B_{k\omega'} \hat{a}_\omega^\dagger \hat{b}_{\omega'}^\dagger \right) |0_{in}\rangle, \quad (107)$$

where  $Z$  is defined by

$$Z^{-2} = |\langle 0_{out} | 0_{in} \rangle|^2. \quad (108)$$

Even though eq. (107) looks cumbersome, one easily verifies that, to order  $\beta^2$ , it correctly gives the relationship between the vacuum decay ( $Z > 1$ ) and the pair creation

probability of Minkowski quanta. Indeed, using  $(F^{-1})_{\omega\omega'} \langle 0_{in} | \hat{a}_{\omega'} \hat{a}_{\omega'}^\dagger | 0_{in} \rangle = \delta_{\omega,\omega'}$  and the condition on  $B_{\omega\omega'}$  and  $F_{\omega\omega'}$  which arises from  $[a_{\omega'}^{out}, b_{\omega'}^{out}] = 0$  and eq. (105), one obtains,

$$Z^2 = 1 + \sum_{\omega\omega'} |B_{\omega\omega'}|^2 + O(\beta^4) . \quad (109)$$

This is the correct expression since the probability to have a pair of *out* quanta is

$$|\langle 0_{out} | a_{\omega}^{out} b_{\omega'}^{out} | 0_{in} \rangle|^2 = \left| \frac{B_{\omega\omega'}}{Z} \right|^2 = |B_{\omega\omega'}|^2 + O(\beta^4) \quad (110)$$

For completeness, we notice that when the scattering is stationary (as it is the case for uniform acceleration and in black hole evaporation), one has

$$B_{\omega\omega'} = \frac{\beta_{\omega}}{\alpha_{\omega}} \delta_{\omega,\omega'} , \quad F_{\omega\omega'} = \delta_{\omega,\omega'} . \quad (111)$$

Since they are diagonal, eq. (107) becomes

$$|0_{out}\rangle = \frac{1}{Z} \exp \left( \sum_{\omega} \frac{\beta_{\omega}}{\alpha_{\omega}} \hat{a}_{\omega}^\dagger \hat{b}_{\omega}^\dagger \right) |0_{in}\rangle \quad (112)$$

thereby recovering the usual diagonal expression governed by the “Unruh” operators  $\hat{a}_{\omega}, \hat{b}_{\omega}$ .

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